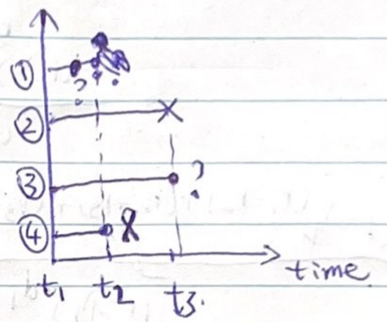
A2002-Q2 (a) $\prod_{j=1}^{k}\left\{\left[\prod_{\ell=1}^{j-1}(1-\lambda_\ell)^{d_j}\right]\lambda_j^{d_j}\left[\prod_{\ell=1}^{j}(1-\lambda_\ell)^{m_j}\right]\right\}$ ①

For one obs • if event $\prod_{\ell=1}^{j-1}(1-\lambda_\ell)\lambda_j$
@ time j • if alive?/censor $\prod_{\ell=1}^{j}(1-\lambda_\ell)$
∴ collectively, @ time j d_j events; m_j censors.
Then multiply ~~ov~~ over j. $\prod_{j=1}^{k}$ [~~over counting?~~]
∴ ☆ $r_{j+1}+d_j+m_j=r_j$. m_j: censor b/w (t_j, t_{j+1})
Let's look at a toy example.

	t_1	t_2	t_3
①	$1-\lambda_1$		
②	$1-\lambda_1$	$1-\lambda_2$	λ_3
③	$1-\lambda_1$	$1-\lambda_2$	$1-\lambda_3$
④	$1-\lambda_1$	λ_2	



t_1 $r_1=4$. $m_1=1$ $d_1=0$
t_2 $r_2=3$ $m_2=0$ $d_2=1$
t_3 $r_3=2$ $m_3=1$ $d_3=1$
$\Rightarrow r_j-r_{j+1}$ ~~$r_{j+1}-r_j$~~ $=d_j+m_j$

r_j: alive/uncesored + death.
$r_j-d_j-m_j \rightarrow r_{j+1}$
☆ ④ died at time 2, but this subject is at risk. ($r_2=3$)
as only it at risk, he/she has the Prob of dying.

① can be rearranged as

$$\prod_{j=1}^{k}\lambda_j^{d_j}\left[\frac{\prod_{\ell=1}^{j}(1-\lambda_\ell)^{d_j}\cdot(1-\lambda_\ell)^{m_j}}{(1-\lambda_j)^{d_j}}\right]$$

$$\prod_{j=1}^{k} \lambda_j^{d_j} \left[\frac{\prod_{\ell=1}^{j} (1-\lambda_\ell)^{d_j+m_j}}{(1-\lambda_j)^{d_j}} \right]$$ ②

where ② can be written as:

$$\frac{\prod_{\ell=1}^{j}(1-\lambda_\ell)^{r_j-r_{j+1}}}{(1-\lambda_j)^{d_j}} = \frac{(1-\lambda_1)^{r_1-r_2}}{(1-\lambda_1)^{d_1}} \cdot \frac{(1-\lambda_1)^{r_2-r_3}(1-\lambda_2)^{r_2-r_3}}{(1-\lambda_2)^{d_2}}$$

$$\cdot \frac{(1-\lambda_1)^{r_3-r_4}(1-\lambda_2)^{r_3-r_4}(1-\lambda_3)^{r_3-r_4}}{(1-\lambda_3)^{d_3}} \cdots \frac{(1-\lambda_1)^{r_j-r_{j+1}}\cdots(1-\lambda_j)^{r_j-r_{j+1}}}{(1-\lambda_j)^{d_j}}$$

$$= \frac{(1-\lambda_1)^{(r_1-r_2)+(r_2-r_3)+(r_3-r_4)\cdots(r_j-r_{j+1})}}{(1-\lambda_1)^{d_1}}$$

$$\times \frac{(1-\lambda_2)^{(r_2-r_3)+(r_3-r_4)+\cdots(r_j-r_{j+1})}}{(1-\lambda_2)^{d_2}}$$

$$\times \frac{(1-\lambda_3)^{(r_3-r_4)+(r_4-r_5)+\cdots(r_j-r_{j+1})}}{(1-\lambda_3)^{d_3}}$$

$$= \frac{(1-\lambda_1)^{r_1-0}}{(1-\lambda_1)^{d_1}} \cdot \frac{(1-\lambda_2)^{r_2-0}}{(1-\lambda_2)^{d_2}} \cdot \frac{(1-\lambda_3)^{r_3-0}}{(1-\lambda_2)^{d_3}} \cdots \frac{(1-\lambda_k)^{r_k}}{(1-\lambda_k)^{d_k}}$$

$$= (1-\lambda_1)^{r_1-d_1} \cdot (1-\lambda_2)^{r_2-d_2} (1-\lambda_3)^{r_3-d_3} \cdots$$

∴ we've shown that survival likelihood can be written as:

$$\prod_{j=1}^{k} \lambda_j^{d_j} (1-\lambda_j)^{r_j-d_j}$$