3 axioms

① $0 \le P(A) \le 1$ A: Event A

② $P(S) = 1,\ P(\phi) = 0$ S: sample space

③ ~~P(U∞~~ $P(\bigcup_{n}^{\infty} A_n) = \sum_{n=1}^{\infty} P(A_n)$ $A_1, A_2 \dots$ are mutually exclusive. ~~E~~ $A_n A_m = \phi$, $n \neq m$

properties

commutativity Distributive laws

① $\overbrace{P(A \cup B)}^{A \text{ or } B} = P(A) + P(B) - P(AB) = P(B \cup A)$ $(A \cup B) \cap C = AC \cup BC$

$P(A \cup B \cup C) = P\{(A \cup B) \cup C\} = P(A \cup B) + P(C) - P\{(A \cup B) \cdot C\}$

A or B or C ... $= P(A) + P(B) - P(AB) + P(C) - \{P(AC) \cup P(BC)\}$

associativity $= P(A) + P(B) + P(C) - P(AB) - \{P(AC) + P(BC) - P(ABC)\}$

$= P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$

By induction, we have:

$$P(\bigcup_{n=1}^{n} E_n) = \sum_{i=1}^{n} P(E_i) - \sum_{\substack{i \neq j \\ i<j}} P(E_i E_j) + \sum_{i<j<k} P(E_{ijk}) + (-1)^{n+1} P(E_1 E_2 \dots E_n)$$

known as inclusion-exclusion identity.

↳ take sum of prob. of events taken one at a time (inclusion)

↳ minus sum of prob of events taken two at a time (exclusion)

↳ plus sum of prob. of .... three at a time (inclusion)

↳ and so on. exclusion

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To summarise:

① commutativity: $P(A \cup B) = P(B \cup A)$

② associativity: $P(A \cup B \cup C) = P\{(A \cup B) \cup C\} = P\{A \cup (B \cup C)\}$

③ distributive law: $P\{(A \cup B) \cap C\} = P(AC \cup BC)$; $P\{(A \cap B) \cup C\} = \cancel{P(A \cup C) \cdot (B \cup C)}$ $P\{(A \cup C) \cap (B \cup C)\}$

④ inclusion-exclusion property.

⑤ ~~A~~ DeMorgan's Laws: $(A \cup B)^c = A^c \cap B^c$; $(A \cap B)^c = A^c \cup B^c$

calculations

① conditional prob. $P(A|B) = \frac{P(AB)}{P(B)}$ $P(AB) = P(A|B)P(B)$

$P(ABC) = \cancel{P(AB|C)}\ P(A|BC)P(BC) = P(A|BC)P(B|C)P(C)$

② independent event $P(A|B) = P(A)$; $P(AB) = P(A)P(B)$

$P(A_1 A_2 \dots A_n) = P(A_1)P(A_2) \dots P(A_n) \Rightarrow A_1, A_2 \dots A_n$ are indep.

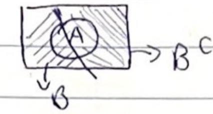
pairwise independent $\nRightarrow$ jointly independent

$P(AB) = P(A)P(B)$ $P(AC) = P(A)P(C)$

$P(BC) = P(B)P(C)$ $\nRightarrow P(A)P(B)P(C) = P(A)P(B)P(C)$

③ Baye's formula

$A = AB \cup AB^C$



philosophy: partition diff problem into pieses.

Total prob. $\quad P(A) = P(AB) + P(AB^C)$

$= P(A|B)P(B) + P(A|B^C)P(B^C)$ $\quad P(B) + P(B^C) = 1$

$= P(A|B)P(B) + P(A|B^C)(1 - P(B))$

more generally:

Total prob. $\quad \cancel{\bigcup_{i=1} B_i} \quad A = \sum_{i=1}^{n} AB_i = \bigcup_{i=1}^{n} AB_i$ where $B_iB_j = \emptyset$ for $i \neq j$ i.e., mutually excusive

$P(A) = P(\cancel{\sum_{i=1}^{n} AB_i}) = \sum_{i=1}^{n} P(AB_i)$

$= \cancel{P(\sum_{i=1}^{n} P} = \sum_{i=1}^{n} P(A|B_i)P(B_i)$

Bayes' formula $\quad$ Bayes' formula:

$$\underbrace{P(B_i|A)}_{\text{posterior dist}} = \frac{P(B_iA)}{P(A)} = \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^{n} P(AB_i)} = \frac{\overbrace{P(A|B_i)}^{\text{likelihood}}\overbrace{P(B_i)}^{\text{Prior}}}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$$

update

conditionally indep. $\quad P(AB|C) = P(A|C)P(B|C)$