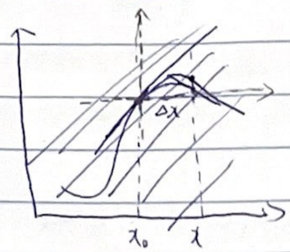
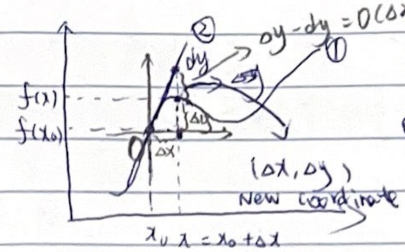
Derivative





The curve:

$\Delta y = f(x) - f(x_0)$

$= f(x_0 + \Delta x) - f(x_0)$

② $(x_0, f(x_0)) \to (x_0 + \Delta x, f(x_0) + \Delta y)$

New coordinate

$O = (x_0, f(x_0))$

$(x, y) \to (\Delta x, \Delta y)$

① $\Delta y = f(x_0 + \Delta x) - f(x_0)$ Change in $f(\cdot)$

$\Delta y = g(\Delta x) \to$ in new coordinate.

② $\Delta y = A\Delta x + O(\Delta x)$

↳ $dy = A\Delta x$ or $dy = A\,dx$ $\qquad$ $dy = A\Delta x$ (straight line in new coordinate ☆

↳ $O(\Delta x) = \underline{\Delta y} - \underline{A\Delta x}$

$\boxed{dy = A\Delta x}$ ☆

Taylor expansion

$$y \approx f(x_0) + \underline{f'(x_0)(x - x_0)} + O(\Delta x) \leftarrow y - f(x_0) = f'(x_0)(x - x_0)$$

$y / f(x)$

$$y = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + R_n(x)$$

1715.OK

增量函数其道

where $R_n(x) = O((x - x_0)^n)$

examples:

e^x: $(e^x)' = e^x$

$$e^x\Big|_{x_0} = e^0 + e^0(x - 0) + \frac{e^0}{2!}(x - 0)^2 + \frac{e^0}{3!}(x - 0)^3 + \dots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + O(x^n)$$