

Results - Real Clinical Study

- **Setting:** clinical trial
- **Patients:** 42 children with acute leukemia
- **Exposure:** 6-MP treatment versus a placebo
- **Outcome:** time to leukemia relapse

Sample 0 (drug 6-MP)	6*, 6, 6, 6, 7, 9*, 10*, 10, 11*, 13, 16, 17*, 19*, 20*, 22, 23, 25*, 32*, 32*, 34*, 35*
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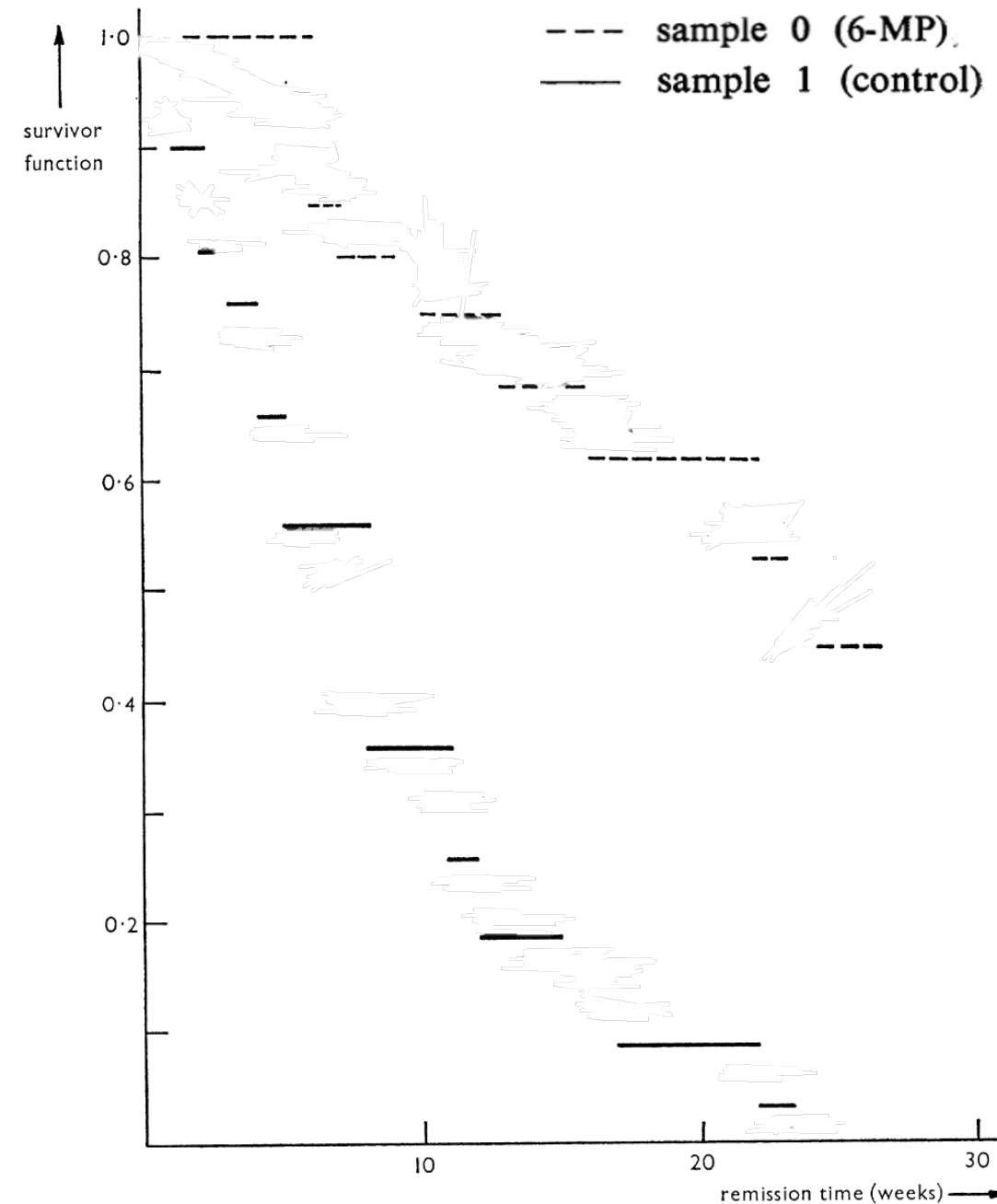
Sample 1 (control)	1, 1, 2, 2, 3, 4, 4, 5, 5, 8, 8, 8, 8, 11, 11, 12, 12, 15, 17, 22, 23
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Note: time in weeks

Results -Kaplan-Meier Curve

Sample 0 (drug 6-MP) 6*, 6, 6, 6, 7, 9*, 10*, 10, 11*, 13, 16, 17*, 19*, 20*, 22, 23, 25*, 32*, 32*, 34*, 35*

Time t_i	Number of events d_i	Number at risk Y_i	Product-Limit Estimator $\hat{S}(t) = \prod_{t_i \leq t} [1 - \frac{d_i}{Y_i}]$
6	3	21	$[1 - \frac{3}{21}] = 0.857$
7	1	17	$[0.857](1 - \frac{1}{17}) = 0.807$
10	1	15	$[0.807](1 - \frac{1}{15}) = 0.753$
13	1	12	$[0.753](1 - \frac{1}{12}) = 0.690$
16	1	11	$[0.690](1 - \frac{1}{11}) = 0.628$
22	1	7	$[0.628](1 - \frac{1}{7}) = 0.538$
23	1	6	$[0.538](1 - \frac{1}{6}) = 0.448$



Results – hypothesis testing and inference

Non-parametric method: Log-rank test

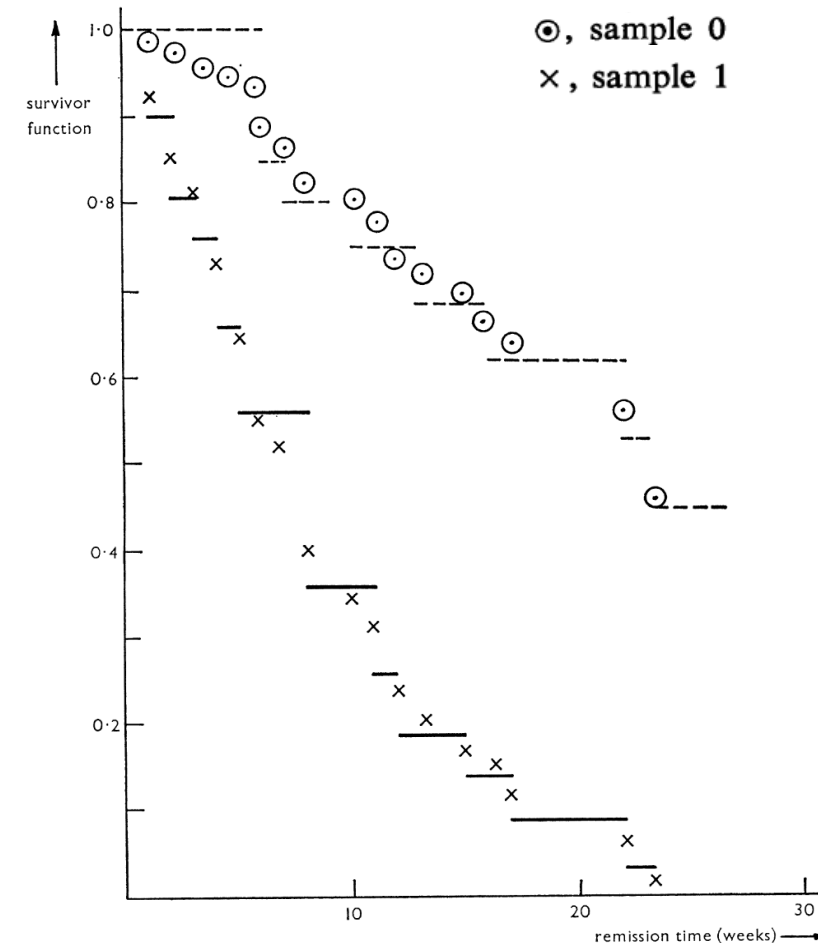
“Failure” time		Risk population		$r_{(i)}$	Multiplicity $A_{(i)}$	$m_{(i)}$
Sample 0	Sample 1	No. in sample 0	No. in sample 1			
23	23	6	1	7	0.1429	2
22	22	7	2	9	0.2222	2
	17	10	3	13	0.2308	1
16		11	3	14	0.2143	1
	15	11	4	15	0.2667	1
13		12	4	16	0.2500	1
	12, 12	12	6	18	0.3333	2
	11, 11	13	8	21	0.3810	2
10		15	8	23	0.3478	1
	8, 8, 8, 8	16	12	28	0.4286	4
7		17	12	29	0.4138	1
	6, 6, 6	21	12	33	0.3636	3
	5, 5	21	14	35	0.4000	2
	4, 4	21	16	37	0.4324	2
	3	21	17	38	0.4474	1
	2, 2	21	19	40	0.4750	2
	1, 1	21	21	42	0.5000	2

$$U(0) = n_1 - \sum m_{(i)} A_{(i)} = 10.25;$$

$$\mathcal{I}(0) = \sum \frac{m_{(i)} \{r_{(i)} - m_{(i)}\}}{r_{(i)} - 1} A_{(i)} \{1 - A_{(i)}\} = 6.2570.$$

Log-rank test statistics: $\frac{U(0)^2}{\mathcal{I}(0)} = 16.79, \chi^2(1), P - \text{value}: 4.18e - 05$

parametric method: Cox regression (under proportionality)



$$\hat{\beta}_{mle} = 1.65, SE = 0.48$$

$$HR = \exp(\hat{\beta}_{mle}) = 5.21 \text{ (95\%CI: 2.18, 13.46)}$$

LR test statistics = 14.90, $\chi^2(1), P - \text{value}: 1.13e - 05$

Graphical Checks of the Proportionality Assumption

1. log-log plot

$$S_{treat}(t) = \exp(-\Lambda_{treat}(t)) = \exp\{-\Lambda_0(t) \exp(\beta)\}$$

$$S_{control}(t) = \exp(-\Lambda_{control}(t)) = \exp\{-\Lambda_0(t)\}$$

Apply log-log transformation:

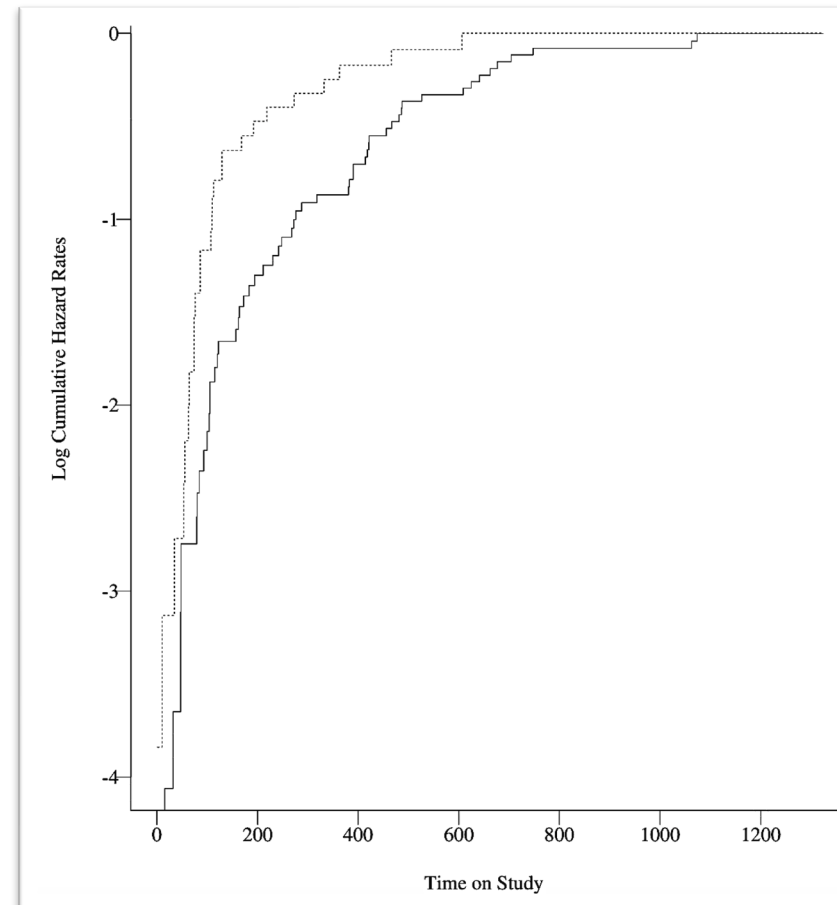
$$\log(-\log(S_{treat}(t))) = \log(\Lambda_{treat}(t)) = \log(\Lambda_0(t)) + \beta$$

$$\log(-\log(S_{control}(t))) = \log(\Lambda_{control}(t)) = \log(\Lambda_0(t))$$

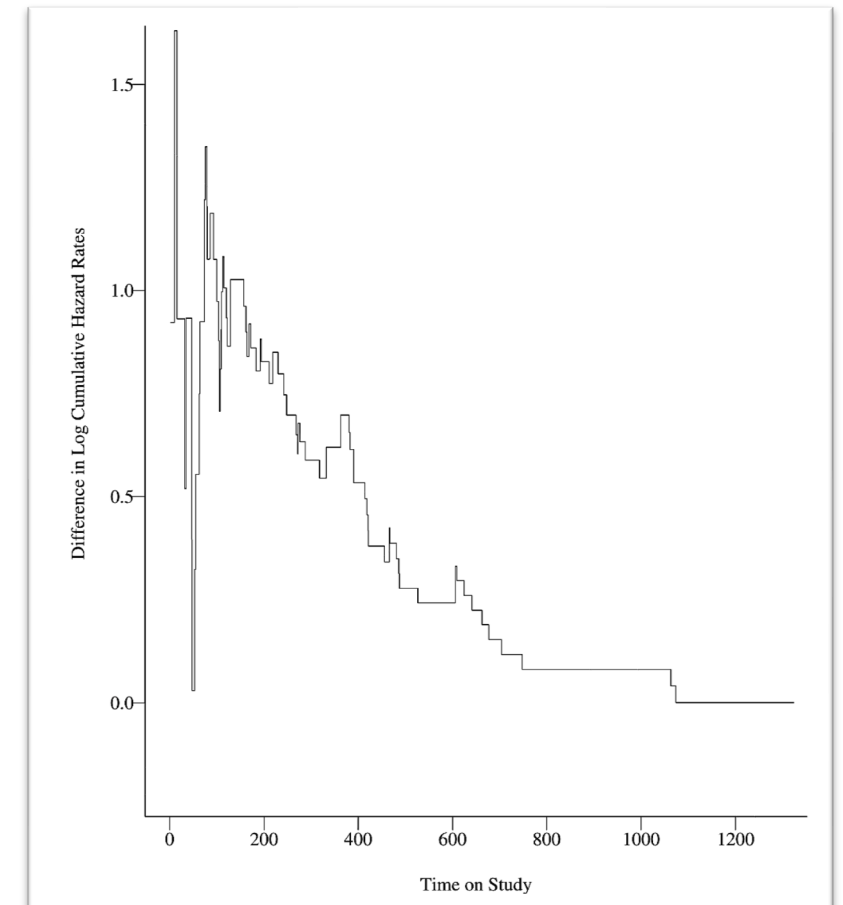
If the proportionality holds:

- log-log plot parallel
- difference in log-log plot horizontal

Log-log plot



Difference in Log-log plot



Graphical Checks of the Proportionality Assumption

2. Andersen plot

Stratified Cox models: each stratum has its own baseline hazard rate

$$\Lambda_i^{treat}(t) = \Lambda_0^{treat}(t) \exp(\beta^T x_i)$$

$$\Lambda_i^{control}(t) = \Lambda_0^{control}(t) \exp(\beta^T x_i)$$

Apply log-log transformation to $S(t) = \exp(-\Lambda(t))$:

$$\log(-\log(S_i^{treat}(t))) = \log(\Lambda_0^{treat}(t)) + \beta^T x_i$$

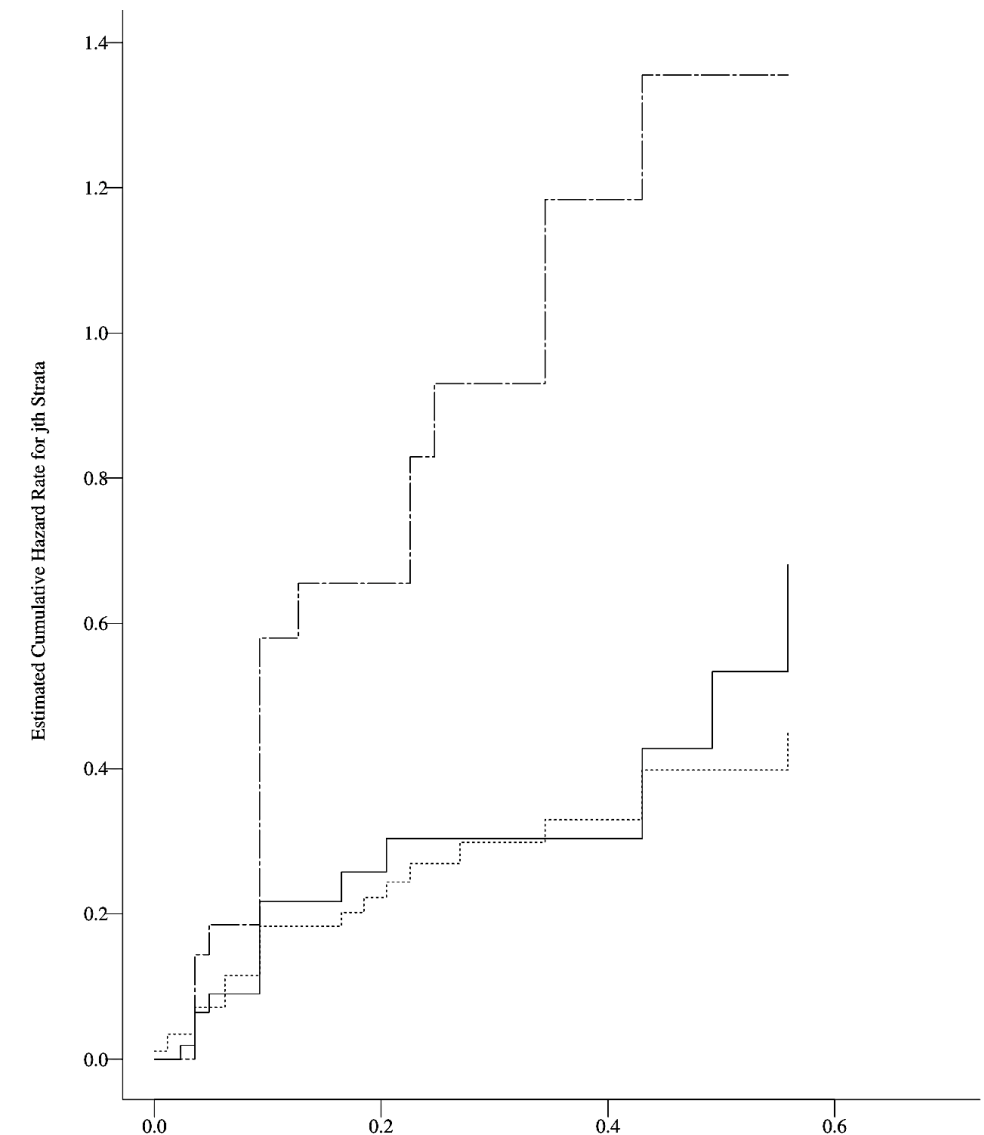
$$\log(-\log(S_i^{control}(t))) = \log(\Lambda_0^{control}(t)) + \beta^T x_i$$

If the proportionality holds:

the cumulative baseline hazard rates among strata

should be a constant

An example of Andersen plot



Graphical Checks of the Proportionality Assumption

3. Schoenfeld residual plot

At given time t , the Schoenfeld residual is defined as:

$$r_k = x_k - E(x_k | Risk\ set)$$

r_k : Schoenfeld residual for the k^{th} covariate.

x_k : the covariate value of individual who experienced event

$E(x_k | Risk\ set)$: the expected value of covariate for these who still at risk

If the proportionality holds:

- residuals scattered around zero

An example of Schoenfeld residual plot

